International Journal of Advanced Research in Engineering Technology & Sciences ISSN: 2394-2819
Feburary-2016 Volume 3, Issue-2
Email: editor@ijarets.org www.ijarets.org

Some Reliability Investigation for a Summer Air Conditioning System with the Boolean Function Expansion Algorithm

Mukesh K. Sharma
Department of Mathematics
R.S.S. (PG), Pilkhuwa, (Hapur)

ABSTRACT:

In this paper, the author has considered a summer air conditioning system for its reliability analysis. Reliability and mean time to failure for considered system have been computed with the aid of Boolean function expansion algorithm. This algorithm is based on the well known theorem of expansion of algebra of logics in conformity. The considered system works as air conditioned for a place in hot and dry weather like Nagpur, Bhopal and Delhi etc. The comfort conditions required in an air conditioned space are 24^0C DBT (dry bulb temperature) and 60% RH (relative humidity)..

KEY WORDS: Algebra of logics

INTRODUCTION:

In the considered system, two air dampers AD_1 and AD_2; two air filters AF_1 and AF_2 are working in parallel redundancy to improve system's overall performance. The input air passes through air dampers, air filters, cooling coils, adiabatic humidifier and water eliminator, and then gives output air. The object of the system is to supply cooled air for selected place. It has been assumed that the failure rates of various components of considered system follow arbitrary distribution and there is no repair facility available for a failed component. The arrangement of equipments required for an ordinary system has been represented in fig-1. The block diagram of the system under considerations has been shown in fig-2

Reliability of the complex system has been computed in case of failure rates follow either Weibull or exponential time distribution. An important reliability parameter, viz; mean time to failure (M.T.T.F.) has also been computed for considered system. A numerical example together with its graphical representation has been appended in last to highlight important results of the study.

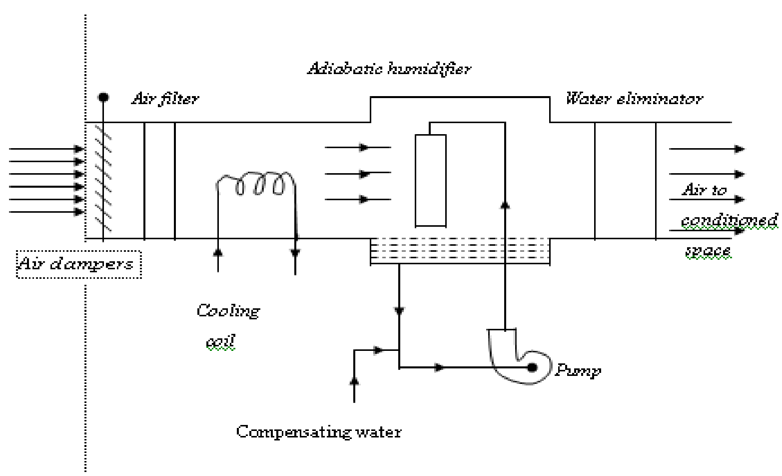

Fig-1 : Summer air conditioning system for hot and dry weather

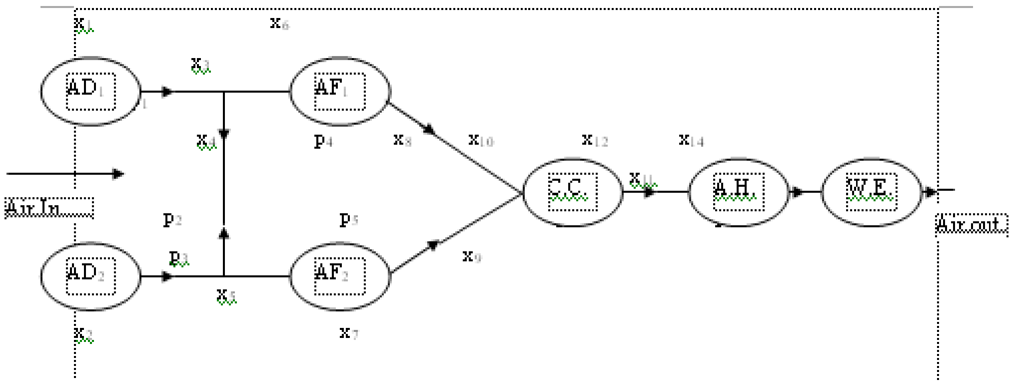

Fig-2 : Block-diagram of considered system

NOTATIONS:

Notations used in this model are as follows:

x_1, x_2	States of air dampers AD_1, AD_2 respectively.
x_3, x_4, x_5	States of pipes P_1, P_2, P_3 respectively.
x_6, x_7	States of air filters AF_1 and AF_2, respectively.
x_{10}, x_{12}, x_{14}	States of cooling coil, air humidifier and water eliminator , respectively.
x_8, x_9, x_{11}, x_{13}	States of pipes P_4, P_5, P_6 and P_7, respectively.
$\wedge / \vee$	Conjunction/Disjunction.
$\cap / \cup$	Intersection/Union.
x_i'	Negation of $x_i, \forall i$
R_i	Reliability of the component corresponding to state x_i.
x_i	=1, in good state; = 0 in bad state.
$P_r(f=1)$	Probability of successful operation of the function *f*.

FORMULATION OF MATHEMATICAL MODEL:

By using Boolean function technique, the conditions of capability of successful operation of the considered system in terms of logical matrix can be expressed as:

$$F(x_1, x_2 --, x_{14}) = \begin{bmatrix} x_1 & x_3 & x_6 & x_8 & x_{10} & x_{11} & x_{12} & x_{13} & x_{14} & \\ x_1 & x_3 & x_4 & x_7 & x_9 & x_{10} & x_{11} & x_{12} & x_{13} & x_{14} \\ x_2 & x_5 & x_7 & x_9 & x_{10} & x_{11} & x_{12} & x_{13} & x_{14} & \\ x_2 & x_4 & x_5 & x_6 & x_8 & x_{10} & x_{11} & x_{12} & x_{13} & x_{14} \end{bmatrix} \qquad \ldots(1)$$

SOLUTION OF THE MODEL:

By using algebra of logics, equation (1) may be written as:

$$F(x_1, x_2 --, x_{14}) = \begin{bmatrix} x_1 & x_3 & x_{10} & x_{11} & x_{12} & x_{13} & x_{14} & \begin{vmatrix} x_6 & x_8 & \\ x_4 & x_7 & x_9 \end{vmatrix} \\ x_2 & x_5 & x_{10} & x_{11} & x_{12} & x_{13} & x_{14} & \begin{vmatrix} x_7 & x_9 & \\ x_4 & x_6 & x_8 \end{vmatrix} \end{bmatrix}$$

$$= x_{10}\ \ x_{11}\ \ x_{12}\ \ x_{13}\ \ x_{14}\left[\begin{array}{ll} x_1 & x_3\left|\begin{array}{lll} x_6 & x_8 & \\ x_4 & x_7 & x_9\end{array}\right. \\ x_2 & x_5\left|\begin{array}{lll} x_7 & x_9 & \\ x_4 & x_6 & x_8\end{array}\right.\end{array}\right]$$

$$= x_{10}\ \ x_{11}\ \ x_{12}\ \ x_{13}\ \ x_{14} f \qquad \ldots(2)$$

where,

$$f = \left[\begin{array}{ll} x_1 & x_3\left|\begin{array}{lll} x_6 & x_8 & \\ x_4 & x_7 & x_9\end{array}\right. \\ x_2 & x_5\left|\begin{array}{lll} x_7 & x_9 & \\ x_4 & x_6 & x_8\end{array}\right.\end{array}\right] \qquad \ldots(3)$$

In equation (3), five arguments (x_4, x_6, x_7, x_8, x_9) are entering twice. Therefore, we can take expansion from any one of them. Let we choose expansion from x_6,

$$f = \left[\begin{array}{l} x_6'\left|\begin{array}{ll} x_1 & x_3\left|\begin{array}{lll} 0 & x_8 & \\ x_4 & x_7 & x_9\end{array}\right. \\ x_2 & x_5\left|\begin{array}{lll} x_7 & x_9 & \\ x_4 & 0 & x_8\end{array}\right.\end{array}\right. \\ x_6\left|\begin{array}{ll} x_1 & x_3\left|\begin{array}{lll} 1 & x_8 & \\ x_4 & x_7 & x_9\end{array}\right. \\ x_2 & x_5\left|\begin{array}{lll} x_7 & x_9 & \\ x_4 & 1 & x_8\end{array}\right.\end{array}\right.\end{array}\right]$$

$$= x_6'\ Y_0 \vee x_7\ Y_1 \qquad \ldots(4)$$

where,

$$Y_0 = \left[\begin{array}{ll} x_1 & x_3\left|\begin{array}{lll} 0 & x_8 & \\ x_4 & x_7 & x_9\end{array}\right. \\ x_2 & x_5\left|\begin{array}{lll} x_7 & x_9 & \\ x_4 & 0 & x_8\end{array}\right.\end{array}\right]$$

$$= \left[\begin{array}{lllll} x_1 & x_3 & x_4 & x_7 & x_9 \\ x_2 & x_5 & x_7 & x_9 & \end{array}\right]$$

$$= \left[\begin{array}{ll} x_7 & x_9\left|\begin{array}{lll} x_1 & x_3 & x_4 \\ x_2 & x_5 & \end{array}\right.\end{array}\right] \qquad \ldots(5)$$

and

$$Y_1 = \left[\begin{array}{ll} x_1 & x_3\left|\begin{array}{lll} 1 & x_8 & \\ x_4 & x_7 & x_9\end{array}\right. \\ x_2 & x_5\left|\begin{array}{lll} x_7 & x_9 & \\ x_4 & 1 & x_8\end{array}\right.\end{array}\right]$$

or,

$$Y_1 = \left[\begin{array}{l} x_1 \quad x_3 \left|\begin{array}{l} x_8 \\ x_4 \quad x_7 \quad x_9 \end{array}\right. \\ x_2 \quad x_5 \left|\begin{array}{l} x_7 \quad x_9 \\ x_4 \quad x_8 \end{array}\right. \end{array}\right] \qquad \ldots(6)$$

All the arguments in equation (5) appear only once, hence Y_0 is non -iterated but the arguments (x_4, x_7, x_8, x_9) enter twice in Y_1, therefore any one of them may be taken to perform further this expansion. Let us conveniently choose x_7 and do expansion as follows:

$$Y_1 = \left[\begin{array}{l} x_7' \left|\begin{array}{l} x_1 \quad x_3 \left|\begin{array}{l} x_8 \\ x_4 \quad 0 \quad x_9 \end{array}\right. \\ x_2 \quad x_5 \left|\begin{array}{l} 0 \quad x_9 \\ x_4 \quad x_8 \end{array}\right. \end{array}\right. \\ x_7 \left|\begin{array}{l} x_1 \quad x_3 \left|\begin{array}{l} x_8 \\ x_4 \quad 1 \quad x_9 \end{array}\right. \\ x_2 \quad x_5 \left|\begin{array}{l} 1 \quad x_9 \\ x_4 \quad x_8 \end{array}\right. \end{array}\right. \end{array}\right]$$

$$= x_7' \; Y_{10} \vee x_7 \; Y_{11} \qquad \ldots(7)$$

where,

$$Y_{10} = \left[\begin{array}{l} x_1 \quad x_3 \left|\begin{array}{l} x_8 \\ x_4 \quad 0 \quad x_9 \end{array}\right. \\ x_2 \quad x_5 \left|\begin{array}{l} 0 \quad x_9 \\ x_4 \quad x_8 \end{array}\right. \end{array}\right]$$

$$= \left[\begin{array}{llll} x_1 & x_3 & x_8 & \\ x_2 & x_5 & x_4 & x_8 \end{array}\right]$$

$$= \left[x_8 \left|\begin{array}{l} x_1 \quad x_3 \\ x_2 \quad x_4 \quad x_5 \end{array}\right. \right] \qquad \ldots(8)$$

and

$$Y_{11} = \left[\begin{array}{l} x_1 \quad x_3 \left|\begin{array}{l} x_8 \\ x_4 \quad x_9 \end{array}\right. \\ x_2 \quad x_5 \left|\begin{array}{l} x_9 \\ x_4 \quad x_8 \end{array}\right. \end{array}\right] \qquad \ldots(9)$$

In equation (8) all the arguments appear once and so Y_{10} is non –iterated. In equation (9) the arguments (x_4, x_8, x_9) appear twice therefore we may take any one of them for further proceedings. Let us choose x_8 for the next expansion. Thus,

$$Y_{11} = \left[\begin{array}{ll} x_8' & \left| \begin{array}{ll} x_1 & x_3 \left| \begin{array}{ll} 0 & \\ x_4 & x_9 \end{array}\right. \\ x_2 & x_5 \left| \begin{array}{ll} x_9 & \\ x_4 & 0 \end{array}\right. \end{array}\right. \\ x_8 & \left| \begin{array}{ll} x_1 & x_3 \left| \begin{array}{ll} 1 & \\ x_4 & x_9 \end{array}\right. \\ x_2 & x_5 \left| \begin{array}{ll} x_9 & \\ x_4 & 1 \end{array}\right. \end{array}\right. \end{array}\right]$$

$$= x_8' \; Y_{110} \vee x_8 \; Y_{111} \qquad \ldots(10)$$

where,

$$Y_{110} = \left[\begin{array}{ll} x_1 & x_3 \left| \begin{array}{ll} 0 & \\ x_4 & x_9 \end{array}\right. \\ x_2 & x_5 \left| \begin{array}{ll} x_9 & \\ x_4 & 0 \end{array}\right. \end{array}\right]$$

$$= \left[\begin{array}{llll} x_1 & x_3 & x_4 & x_9 \\ x_2 & x_5 & x_9 & \end{array}\right]$$

$$= \left[\begin{array}{ll} x_9 & \left| \begin{array}{lll} x_1 & x_3 & x_4 \\ x_2 & x_5 & \end{array}\right. \end{array}\right] \qquad \ldots(11)$$

and

$$Y_{111} = \left[\begin{array}{ll} x_1 & x_3 \left| \begin{array}{ll} 1 & \\ x_4 & x_9 \end{array}\right. \\ x_2 & x_5 \left| \begin{array}{ll} x_9 & \\ x_4 & 1 \end{array}\right. \end{array}\right]$$

$$= \left[\begin{array}{ll} x_1 & x_3 \\ x_2 & x_5 \left| \begin{array}{l} x_9 \\ x_4 \end{array}\right. \end{array}\right] \qquad \ldots(12)$$

In equations (11) and (12), all the arguments appear once, hence both functions Y_{110} and Y_{111} are non-iterated and are not subjected to further expansion. Hence equation (4) gives, by using equations (5) through (12):

$$f = \left[\begin{array}{ll} x_6' & Y_0 \\ x_6 & \left| \begin{array}{ll} x_7' & Y_{10} \\ x_7 & \left| \begin{array}{ll} x_8' & Y_{110} \\ x_8 & Y_{111} \end{array}\right. \end{array}\right. \end{array}\right]$$

$$=\begin{bmatrix} x_6' & Y_0 & & \\ x_6 & x_7' & Y_{10} & \\ x_6 & x_7 & x_8' & Y_{110} \\ x_6 & x_7 & x_8 & Y_{111} \end{bmatrix} = \begin{bmatrix} H_1|Y_0 \\ H_2|Y_{10} \\ H_3|Y_{110} \\ H_4|Y_{111} \end{bmatrix} \qquad \ldots(13)$$

where, $H_1 = x_6'$, $H_2 = x_6 x_7'$, $H_3 = x_6 x_7 x_8'$ and $H_4 = x_6 x_7 x_8$.

Clearly H_1, H_2, H_3 and H_4 are pair-wise disjoint.

$$\therefore P_r(f=1) = \sum_{i=1}^{4} P_r(H_i).P_r\left(\frac{f}{H_i}\right) \qquad \ldots(14)$$

or, $P_r(f=1) = P_r(x_6').P_r(z_1) + P_r(x_6 x_7').P_r(z_2) + P_r(x_6 x_7 x_8').P_r(z_3)$
$+ P_r(x_6 x_7 x_8).P_r(z_4)$ …(15)

Where, $z_1 = Y_0$, $z_2 = Y_{10}$, $z_3 = Y_{110}$ *and* $z_4 = Y_{111}$ and the events $H_i(\forall i = 1,2,3,4)$ form a complete group of incompatible hypothesis. Then $P_r\left(f/H_i\right)$ form the conditional probability of a good state of the system for each hypothesis.

Now, if R_i be the reliability corresponding to component state x_i, then equation (14) gives:

$$\begin{aligned} P_r(f=1) &= (1-R_6)R_7R_9\{1-(1-R_1R_3R_4)(1-R_2R_5)\} \\ &\quad + R_6(1-R_7)R_8\{1-(1-R_1R_3)(1-R_2R_4R_5)\} \\ &\quad + R_6R_7(1-R_8)R_9\{1-(1-R_1R_3R_4)(1-R_2R_5)\} \\ &\quad + R_6R_7R_8[1-(1-R_1R_3)\times\{1-R_2R_5(1-(1-R_9)(1-R_4))\}] \qquad \ldots(16) \end{aligned}$$

Finally, the probability of successful operation, i.e., reliability of the considered system is given by:

$$\begin{aligned} R_S = P_r(F=1) &= P_r(x_{10}x_{11}\,x_{12}x_{13}\,x_{14}).P_r(f=1) \\ &= R_{10}R_{11}R_{12}R_{13}R_{14}[R_2R_5R_7R_9 + R_1R_3R_4R_7R_9 + R_1R_3R_6R_8 + R_2R_4R_5R_6R_8 \\ &\quad + R_1R_2R_3R_4R_5R_6R_7R_8R_9 + R_1R_2R_3R_4R_5R_6R_7R_8R_9 - R_1R_2R_3R_4R_5R_7R_9 \\ &\quad - R_1R_2R_3R_4R_5R_6R_8 - R_1R_3R_4R_6R_7R_8R_9 - R_2R_4R_5R_6R_7R_8R_9 \\ &\quad - R_1R_2R_3R_5R_6R_7R_8R_9] \qquad \ldots(17) \end{aligned}$$

PARTICULAR CASES:

CASE I*: *When reliability of each component of system is R:

In this case, equation (17) yields:

$$\begin{aligned} R_S &= R^5\left[2R^4 + 4R^5 - 2R^7 - R^8 + 2R^9\right] \\ &= R^9\left[2 + 2R - 4R^3 - R^4 + 2R^5\right] \qquad \ldots(18) \end{aligned}$$

CASE II*: *When failure rates follow weibull distribution:

In this case, the reliability of considered system is given by:

$$\mathrm{R_{SW}}(t) = \sum_{i=1}^{6} \exp.\{-a_i t^\alpha\} - \sum_{j=1}^{5} \exp.\{-b_j t^\alpha\} \qquad \ldots(19)$$

where α is a real positive parameter and

$a_1 = c + \lambda_2 + \lambda_5 + \lambda_7 + \lambda_9$

$a_2 = c + \lambda_1 + \lambda_3 + \lambda_4 + \lambda_7 + \lambda_9$

$a_3 = c + \lambda_1 + \lambda_3 + \lambda_6 + \lambda_8$
$a_4 = c + \lambda_2 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_8$
$b_1 = c + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_7 + \lambda_9$
$b_2 = c + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_8$
$b_3 = c + \lambda_1 + \lambda_3 + \lambda_4 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9$
$b_4 = c + \lambda_2 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9$
and $c = \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{13} + \lambda_{14}$

Where λ_i be the failure rate corresponding to system state x_i.

CASE III*: *When failure rates follow exponential distribution:

Exponential time distribution is the particular case of weibull time distribution for $\alpha = 1$ and is very useful in numerous practical problems. Therefore the reliability function for the whole system at time instant 't', is given by:

$$\mathrm{R}_{\mathrm{SE}}(t) = \sum_{i=1}^{6} \exp.\{-a_i t\} - \sum_{j=1}^{5} \exp.\{-b_j t\} \qquad \ldots(20)$$

where $a_i's$ and $b_j's$ have been mentioned earlier.

Also, in this case, an important reliability parameter M.T.T.F , is given by

$$M.T.T.F. = \int_0^\infty R_{SE}(t)dt = \sum_{i=1}^{6}\left(\frac{1}{a_i}\right) - \sum_{j=1}^{5}\left(\frac{1}{b_j}\right) \qquad \ldots(21)$$

NUMERICAL COMPUTATION:

For a numerical computation, let us consider the values:

(i) $\lambda_i(i = 1,2,--14) = \lambda = 0.001$, $\alpha = 2$ and $\mathrm{t} = 0,1,2---$. Using these values in equation (19), we compute the table -1.

(ii) $\lambda_i(i = 1,2,--14) = \lambda = 0.001$ and $\mathrm{t} = 0,1,2---$. Using these values in equation (20), we compute the table-1.

(iii) Putting $\lambda_i(i = 1,2,--14) = \lambda = 0.001, 0.002\ ---0.01$ in equation (21), we compute table-2.

Table-1

t	$R_{SW}(t)$	$R_{SE}(t)$
0	1	1
1	0.995005	0.995005
2	0.980074	0.990018
3	0.955384	0.985041
4	0.921259	0.980074
5	0.878210	0.975116
6	0.826981	0.970168
7	0.768586	0.965230
8	0.704330	0.960302
9	0.635794	0.955384
10	0.564783	0.950476

Table-2

λ	$M.T.T.F.$
0	∞
0.001	154.823
0.002	77.41148
0.003	51.60765
0.004	38.70574
0.005	30.96459
0.006	25.80383
0.007	22.11756
0.008	19.3587
0.009	17.20255
0.010	15.48230

DISCUSSION:

Analysis of table-1 reveals that value of reliability function decreases rapidly, in case, failures follow Weibull time distribution but it decreases merely in constant manner for exponential time distribution. Therefore, reliability function remains better in case of exponential time distribution. A critical examination of table-2 concludes that M.T.T.F. Decreases catastrophically for the lower values of failure rate λ but it decreases smoothly for higher values of λ.

REFERENCES:

1. Dhillon, B. S.; Natesan, J.: "Stochastic Analysis of Outdoor Power System in Fluctuating Environment", Microelectronic Reliab. , Vol.23, pp 867 – 887, 1983.
2. Gupta, P.P.; Sharma, R.K.: "Cost Analysis of Three-state Standby Redundant Electronic Equipment", Microelectronic Reliab. , Vol. 25, pp 1029-1033, 1985.
3. Jain, S.P.; Gopal, K.: "A Heuristic Method of Link Reliability Assignment for Maximal Reliability", Microelectronic Reliab. , Vol. 30, pp 673 – 679, 1990.
4. Jen – Shyan, W.; Rong – Jaye, C.: "Efficient Algorithm for Reliability of a Circular Consecutive k-out-of-n: F System", IEEE TR on Reliability, Vol. 55, issue – 1, pp 163- 164, 2006.
5. Kuo, W; Prasad, V.R.; Tillman, F.A. and Hwang, C.L: "*Fundamental and Applications of Reliability Optimization*", Cambridge University Press, Cambridge, 2000.
6. Sharma,M.K., Bansal K.K.: A Comparative Study of Reliability Analysis of a Non-Series Parallel Network. International Journal of Education and Science Research Review.Volume-2 Issue-6 2015
7. Nagraja, H.N.; Kannan, N.; Krishnan, N.B.: "*Reliability*", Springer Publication, 2004.
8. Sharma, Deepankar : "Cost analysis, availability and MTTF of a complex system with the concept of waiting and human error", Bulletin of pure and applied sciences, Vol. 21 E, No. 2, pp 553-559 (2002).
9. Singh, J. ; Goyal, P. : "Availability analysis of a standby complex system having imperfect switch over device", Microelectron. Reliab. 35, pp 285-288 (1996).
10. Vanderhaegen, F.; Jouglet, D.; Piechowiak, S.: "Human Reliability Analysis of Co-operative Redundancy to Support Diagnosis", IEEE TR. on Reliability, Vol. 53, issue-4, pp 458-464, 2004
11. Zhimin, He; Han, T.L.; Eng., H.O.: "A Probabilistic Approach to Evaluate the Reliability of Piezoelectric Micro-Actuators", IEEE TR. on Reliability, Vol. 54, issue-1, pp 83-91, 2005.